A Circular-Electric Hybrid Junction and Some Channel-Dropping Filters

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A $TE_{01}°$ hybrid junction that operates similarly to the Riblet short-slot hybrid is described, but because the modes involved are circular-electric, the hybrid can be telescopically mounted, allowing for adjustment to almost any power division. The experimental results show that, centered at 55.6 kmc, the frequency range is larger than 20 per cent. Adjusted for equal power division, the balance is better than 0.5 db and the unwanted reflections in the driven and balanced (isolation) arms are at least 23 db below the input signal.

Using the hybrid together with band-reflection, band-transmission or high-pass filters, it is possible to build low-loss channel-dropping filters. In particular, the use of simple cutoff waveguides permits the design of filters with almost rectangular transfer characteristics.

I. INTRODUCTION

The importance of hybrid junctions for many purposes — measuring, filtering, balancing, equalizing, etc. — need hardly be emphasized. The long distance waveguide communication system[1] operating with the low-loss circular-electric $TE_{01}°$ mode has only two hybrids available: the directional coupler, which has a fixed power division, and the optical hybrid,[2] which requires multimode waveguides. This paper describes a third hybrid, which operates like Riblet's coupler[3] and which adds to the well-known advantages of that coupler the unique property of adjustable power division.

Adjusted for 3-db power division, the hybrid, together with mode-conversion band-rejection filters,[4] band transmission filters or cutoff waveguides,[2] can be used as low-loss components of constant-resistance channel-dropping filters.[5] The scheme that uses high-pass filters (cut-off waveguides) deserves special attention because the amplitude transfer characteristic of the dropped channel can be made to approach a

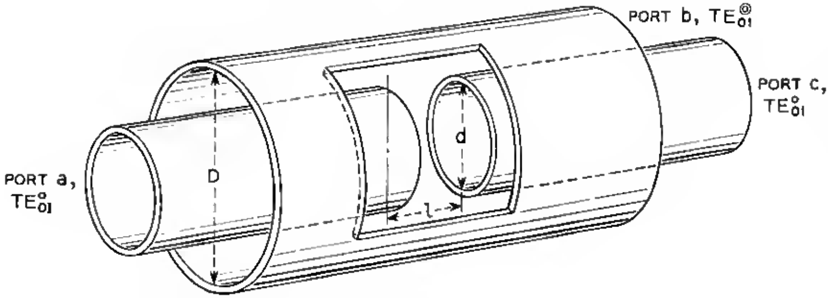


Fig. 1 — Circular-electric hybrid.

rectangular shape of arbitrary bandwidth. This permits not only relaxing the demands on the guard bands between neighboring channels, but also the multiplexing of bands too broad (extremely short pulses)[6] to be handled by mode-conversion filters.

II. DESCRIPTION OF THE HYBRID

The hybrid consists of two coaxial circular metallic tubes, of which the inner one has a gap l, as shown in Fig. 1. The ratio of diameters selected is equal to the ratio of the second to the first roots of the Bessel function J_1 :

$$\frac{D}{d} = \frac{7.016}{3.832} = 1.831. \tag{1}$$

The outer diameter D is chosen so that it cuts off the TE_{03}° mode at the highest frequency of design of the hybrid.

The hybrid is made of two four-port junctions like the one of Fig. 2. It will be shown that power entering in any port is almost equally divided between the two forward modes. Consequently, going back to

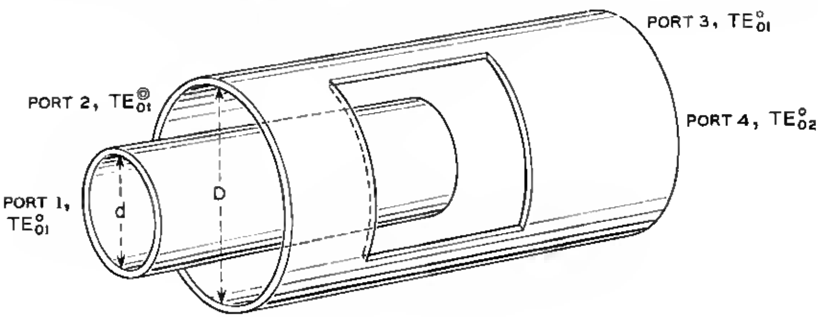


Fig. 2 — Four-port junction.

Fig. 1, power entering in any port is almost equally divided between $TE_{01}°$ and $TE_{02}°$ in the gap region. Each one of these modes repeats the power division at the end of the gap, so the power collected in each output depends on the relative phases of the modes at the end of the gap. Since the velocities of these modes are different, the relative phase, and consequently the power division, can be selected arbitrarily by changing the length of the gap.

III. PROPERTIES OF THE FOUR-PORT JUNCTION AND THE HYBRID

The most general scattering matrix for the reciprocal four-port device of Fig. 2 is

$$S = \begin{vmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{vmatrix}, \tag{2}$$

Entering port 4 with mode $TE_{02}°$, since the metallic inner tube has its surface where the electric field is zero (first zero of the J_1 function), the boundary conditions are automatically satisfied, the $TE_{02}°$ mode is unperturbed, and consequently the back-scattering

$$S_{34} = S_{44} = 0. \tag{3}$$

Furthermore, the forward-scattering coefficients at the plane where the coaxial waveguide starts are

$$S_{14} = \left| \frac{\int_0^{3.832} J_1^2(\alpha)\alpha d\alpha}{\int_0^{7.016} J_1^2(\alpha)\alpha d\alpha} \right|^{\frac{1}{2}} = 0.733, \tag{4}$$

$$S_{24} = - \left| \frac{\int_{3.832}^{7.016} J_1^2(\alpha)\alpha d\alpha}{\int_0^{7.016} J_1^2(\alpha)\alpha d\alpha} \right|^{\frac{1}{2}} = -0.68. \tag{5}$$

Assuming the junction to be nondissipative, (2) must satisfy, because of conservation of energy, the following unitary relations[7]

$$\sum_{\beta=1}^{4} S_{\beta m} S_{\beta n}^* = \begin{cases} 1 & \text{if} \quad m = n \\ 0 & \text{if} \quad m \neq n \end{cases} \tag{6}$$

in which the asterisk means "complex conjugate of."

From (3) and (6),

$$S_{12} = -\frac{S_{14}^*}{S_{24}^*} S_{11}, \tag{7}$$

$$S_{22} = \left(\frac{S_{14}^*}{S_{24}^*}\right)^2 S_{11}, \tag{8}$$

$$S_{13} = | S_{24} | \left(1 - \left|\frac{S_{11}}{S_{24}^2}\right|^2\right)^{\frac{1}{2}} e^{i\theta_{13}}, \tag{9}$$

$$S_{23} = -\frac{S_{14}^*}{S_{24}^*} | S_{24} | \left(1 - \left|\frac{S_{11}}{S_{24}^2}\right|^2\right)^{\frac{1}{2}} e^{i\theta_{13}}, \tag{10}$$

$$S_{33} = -\frac{S_{11}^*}{| S_{24} |^2} e^{i2\theta_{13}}, \tag{11}$$

where θ_{13} is the phase of S_{13}.

Since S_{14} and S_{24} are known from (4) and (5), the five previous expressions become

$$S_{12} = 1.078\, S_{11}, \tag{12}$$

$$S_{22} = 1.163\, S_{11}, \tag{13}$$

$$S_{13} = 0.68(1 - 4.68 \mid S_{11} \mid^2)^{\frac{1}{2}} e^{i\theta_{13}}, \tag{14}$$

$$S_{23} = 0.733(1 - 4.68 \mid S_{11} \mid^2)^{\frac{1}{2}} e^{i\theta_{13}}, \tag{15}$$

$$S_{33} = -2.163\, S_{11}^* e^{i2\theta_{13}}. \tag{16}$$

In the experimental hybrid to be described later on, the modulus of the reflection coefficient is

$$| S_{11} | < 0.05$$

and consequently powers of S_{11} bigger than one can be neglected. With this simplification, the forward transfer elements of the scattering matrix of the hybrid (Fig. 1) are

$$S_{ac} = S_{14}^2 e^{i2\pi l/\lambda_{g2}} \left[1 + \left|\frac{S_{13}}{S_{14}}\right|^2 e^{i2\theta_{13} + i2\pi l/\Lambda}\right], \tag{17}$$

$$S_{ab} = S_{14} S_{24} e^{i2\pi l/\lambda_{g2}} [1 - e^{i2\theta_{13} + i2\pi l/\Lambda}], \tag{18}$$

where

$$\Lambda = \frac{\lambda_{g1}\lambda_{g2}}{\lambda_{g2} - \lambda_{g1}} \tag{19}$$

is the beating wavelength between TE_{01}° and TE_{02}° in the gap;

$$\lambda_{g1} = \frac{\lambda}{\sqrt{1 - \left(\frac{3.832\lambda}{\pi D}\right)^2}} \tag{20}$$

and

$$\lambda_{g2} = \frac{\lambda}{\sqrt{1 - \left(\frac{7.016\lambda}{\pi D}\right)^2}} \tag{21}$$

are the TE_{01}° and TE_{02}° guided wavelengths; and λ is the free-space wavelength.

For a given gap l, the power division of the hybrid K, and the phase shift between the two outputs are derived from (4), (5), (14), (15), (17) and (18):

$$K = \left|\frac{S_{ac}}{S_{ab}}\right|^2 = \frac{1.011 + \cos\left(2\theta_{13} + \frac{2\pi l}{\Lambda}\right)}{1 - \cos\left(2\theta_{13} + \frac{2\pi l}{\Lambda}\right)}, \tag{22}$$

$$\theta_{ac} - \theta_{ab} = \pm\pi + tg^{-1} 13.15 \sqrt{K - 0.0055}. \tag{23}$$

The possible range of power division K obtained from (22) is

$$0.0055 \leqq K < \infty. \tag{24}$$

For $K = 0.0055$, the power flowing in the inner guide is a minimum and specifically 26 db below the input. For $K = \infty$, the power flowing in the coaxial guide is zero.

Since the beating wavelength Λ, as well as the argument θ_{13}, are frequency-sensitive, the power division K given in (22) also varies with frequency. We have not calculated θ_{13}, but it is known[3] that the frequency dependence of θ_{13} and of Λ tend to cancel each other's effect, allowing the power division K to be constant over a relatively broad band. Furthermore, it is very easy to adjust experimentally the gap l for any allowable power division K because the hybrid can be built with sliding tubes. The modes involved are circular electric and consequently the cracks do not interrupt conduction current lines.

IV. EXPERIMENTAL RESULTS

In order to make available the power from the hybrid a four-port transducer has been electroformed capable of transferring TE_{01}° to

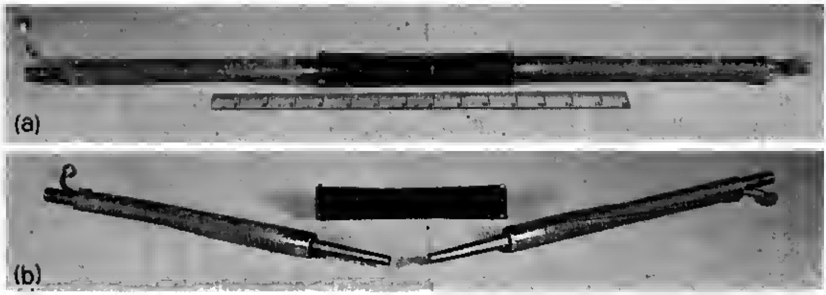


Fig. 3 — (a) Circular-electric hybrid assembled with $TE_{01}^{\odot}$ to $TE_{10}^{\square}$ transducers; (b) exploded view.

TE_{01}° and $TE_{01}^{\odot}$ to $TE_{10}^{\square}$ (Fig. 3). The last change of modes is obtained by smoothly deforming a rectangular waveguide into a coaxial waveguide. The transducer generates small amounts of unwanted higher order modes, which can resonate[8] and ruin the behavior of the hybrid. The resonances can be damped by using for the external tube of the hybrid a lossy-jacket helix waveguide,[9] which substantially attenuates any mode with axial conduction currents.

Fig. 4 shows the electrical behavior of the hybrid adjusted for equal power division. From 50 to 61.2 kmc the balance is better than 0.5 db and the isolation better than 23 db.

At 55.6 kmc the power lost in the hybrid and transducer is 0.83 db. In order to prove that most of this loss occurs in the $TE_{01}^{\odot}$ to $TE_{10}^{\square}$ transducer, the gap was enlarged until, at 55.5 kmc, most of the power was recovered in the inner waveguide ($K = \infty$; $l = 0.906$ inch). The measured insertion loss was then reduced to 0.3 db.

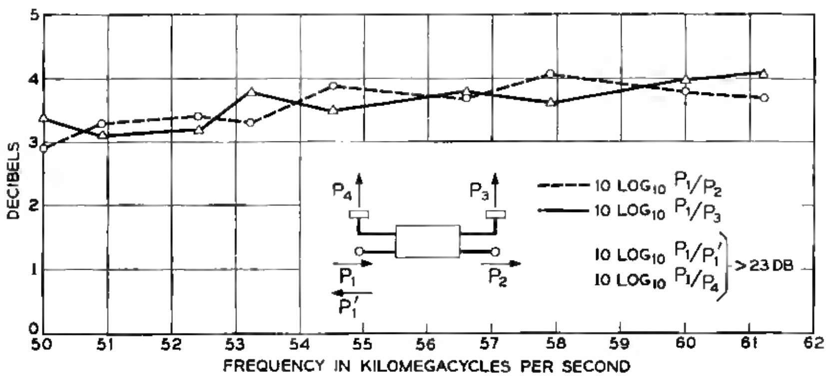


Fig. 4 — Performance of circular-electric hybrid and $TE_{01}^{\odot}$ to $TE_{10}^{\square}$ transducers.

No efforts have been made to improve either the hybrid or the transducers. The possible changes for the hybrid are of an experimental nature and consist in varying the diameter of the gap region and including circular symmetric lumped discontinuities to improve the balance and decrease the unwanted reflections. The possible improvement of the transducer consists in passing from the relatively simple-to-build linear taper used in these experiments to more sophisticated designs[10] that reduce mode conversion.

V. CONSTANT-RESISTANCE CHANNEL-DROPPING FILTERS

It is known that a constant-resistance channel-dropping filter[5] (input matched at all frequencies) can be made using two hybrids connected by two filtering paths. The hybrid described in Section III lends itself to use with filters that operate with low-loss circular-electric modes, and is consequently attractive for use in the long distance waveguide communication system.

The filters that most naturally suit the hybrid are those that possess circular symmetry. For example, filters made with inductive irises, mode-conversion filters[4] and cutoff filters. In Fig. 5, two such filters with identical transfer characteristics are located symbolically in the inner and outer waveguides connecting two circular-electric hybrids. TE_{01}° power that enters port 1, and is rejected by the filters, recombines as $TE_{01}^{\odot}$ in port 2. The power transmitted through the filters can be made to recombine either in port 3 or in port 4. On one hand, assuming the gaps of both hybrids to be identical, power recombines in port 3 if the inner and outer electrical paths between planes a and b are identical, and power recombines in port 4 if those paths differ by π radians. On the other hand, assuming the two paths to be identical,

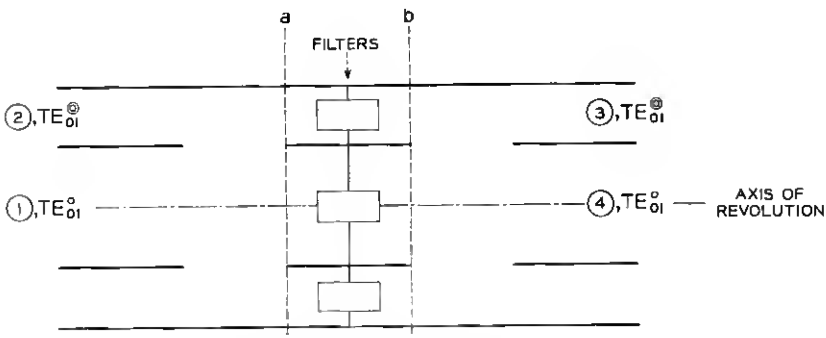


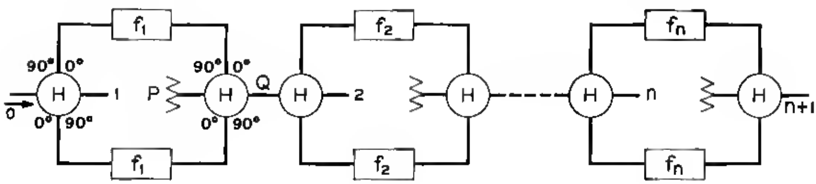
Fig. 5 — Channel-dropping filter using circular-electric hybrids

power recombines in port 3 if the hybrids are identical and recombines in port 4 if both gaps differ by half a beating wavelength between the $TE_{01}°$ and $TE_{02}°$ modes.

Probably the most interesting of the channel-dropping filters is obtained by using cutoff waveguides (high-pass filters) in the connecting paths. The interest comes from the fact that the transfer characteristic of the dropped channel can be made to approximate a rectangular shape.

Before considering the actual geometry of these filters we analyze the behavior of a chain of constant resistance filters represented symbolically in Fig. 6(a). The first link consists of two hybrids H connected by two paths of identical transfer and reflection coefficients. Each path includes a high pass filter that cuts off at frequency f_1. The only difference between the successive constant resistance filters is the cutoff frequency of the high-pass filters. Because of the phase-shifts between the different arms of the hybrids, and the similitude of the connecting paths, power entering in port 0 can be recovered only in ports 1, 2, 3 $\cdots$ $(n + 1)$. The power transfers between input and output ports are given in Fig. 6(b); $n - 1$ channels can be dropped out of n constant resistance filters.

The actual geometry of one of the units of the chain is very simple



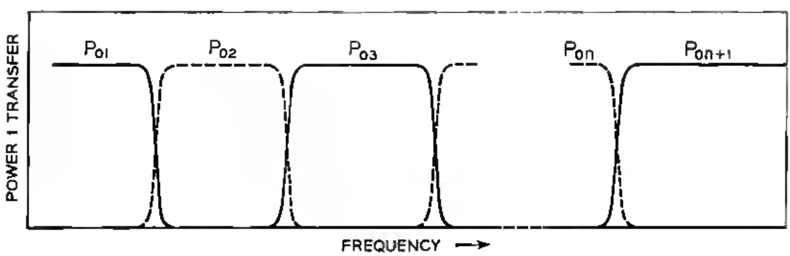


Fig. 6 — (a) Chain of constant-resistance filters; (b) power transfer between ports 0 and 1, 2, 3, $\cdots$, $(n + 1)$.

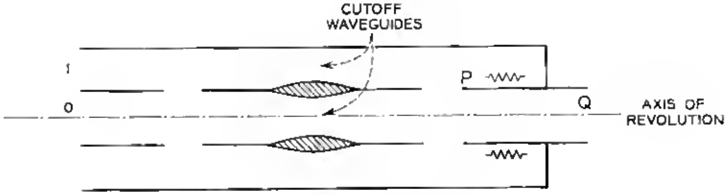


Fig. 7 — Constant-resistance filter.

when circular-electric hybrids and cutoff waveguides are used as shown in Fig. 7. The ports 0, 1, P and Q correspond to those of the first unit in Fig. 6(a). The two hybrids in Fig. 7 are different, in order to recombine the power transmitted through the cutoff sections in the inner waveguide.

Without cavities we have achieved an almost rectangular transfer characteristic of arbitrary width. The guard band between successive channels can be made, at least in principle, arbitrarily small. A working model of cutoff filters has been demonstrated in Ref. 2.

There is another channel-dropping filter worth considering because of its simplicity and because it uses the structure shown in Fig. 2 as a hybrid. [The reader can check that the scattering coefficients of this junction given in (4), (5), (12), (13), (14), (15) and (16) are very close to those of a hybrid when S_{11} is negligibly small.]

Before considering the actual channel-dropping filter we shall describe a microwave equivalent circuit, Fig. 8. It consists of two hybrids, indicated as Riblet couplers, which are connected by two waveguides of equal electrical lengths. These waveguides are also coupled through two identical resonating cavities. The electrical distance between coupling holes in the upper waveguide is an odd multiple of π and in the lower waveguide is an even multiple of π.

Out of resonant frequency of the cavities power entering port 1 splits in equal parts in the first hybrid and recombines in port 5 of the second

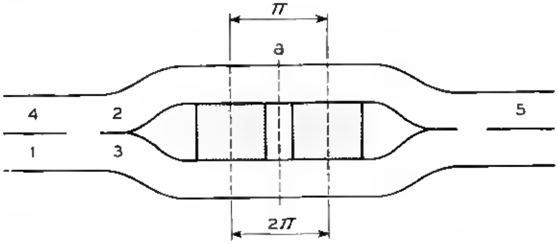
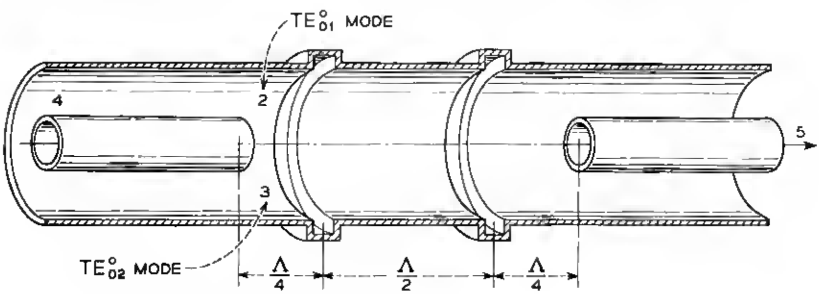


Fig. 8 — Microwave equivalent circuit of channel-dropping filter of Fig. 9.

hybrid. At resonance, power entering port 1 splits in equal parts and each one excites the cavities in different ways. Let us follow the power in the upper path. Because of the distance between coupling holes, the cavities are excited in opposite phase and the reradiation from the cavities is such that all the power flows back toward port 2 as if reflected from an equivalent short circuit located in plane of symmetry a. Meanwhile, the power flowing in the lower path from port 3 excites the cavities in phase and again, because of the adequate distance between holes, all the power goes back toward port 3 as if reflected by a short circuit in plane a. Recombination of the two waves reflected in plane a takes place in port 4 of the first hybrid.

The actual microwave circuit for circular electric waves is shown in Fig. 9. The two hybrids are like those of Fig. 2. Waves flowing in ports 2 and 3 of Fig. 8 are equivalent to the TE_{01}° and TE_{02}° waves in the gap of Fig. 9. The length of the gap region is one beating wavelength between the TE_{01}° and the TE_{02}° modes; the diameter is selected in such a way that the TE_{03}° is cut off except for two enlarged regions where resonance of this mode takes place.[4] These mode-conversion resonant "cavities" couple to both TE_{01}° and TE_{02}° modes and are separated by half a guided wavelength measured in TE_{02}° mode and one guided wavelength measured in TE_{01}° mode. The mode conversion "cavities" are therefore equivalent to the resonant cavities of Fig. 8.

If the coupling between TE_{03}° and TE_{01}° is different from the coupling between TE_{03}° and TE_{02}°, the channel-dropping filter no longer has constant resistance. This can be deduced from Fig. 8 by making the coupling holes in the upper waveguide different from those in the lower one.



$$\Lambda = \frac{\lambda_{g_1}\lambda_{g_2}}{\lambda_{g_2}-\lambda_{g_1}} = \text{BEATING WAVELENGTH BETWEEN } TE_{01}^\circ \text{ AND } TE_{02}^\circ \text{ WAVES}$$

Fig. 9 — Channel-dropping filter with TE_{03}° mode-conversion filter.

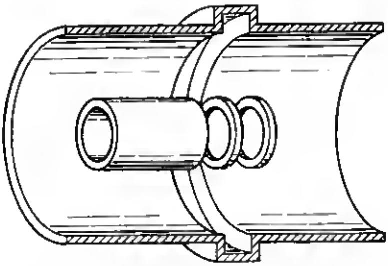


Fig. 10 — Rings to equalize coupling between TE_{03}° and TE_{01}° and between TE_{03}° and TE_{02}°.

To equalize the couplings in Fig. 9, rings like those shown in Fig. 10 can be used.

In all the filters described in this section, the dropped channel appears as $TE_{01}^{\circledcirc}$ mode. It may be necessary to transduce this mode into $TE_{10}^{\square}$. There are essentially two techniques. One consists in using a broadband transducer like the one described in Fig. 3 of Section IV; another consists in using a transmission cavity that resonates with coaxial circular electric mode and that couples to the coaxial waveguide and to a rectangular waveguide.[11] The second approach yields a much shorter transducer but it is not broadband.

VI. CONCLUSIONS

A hybrid capable of dividing TE_{01}° mode into TE_{01}° and $TE_{01}^{\circledcirc}$ has been described. It operates similarly to the Riblet short-slot hybrid, but because the modes involved are circular electric, the hybrid can be made of sliding coaxial tubes that allow adjustment to almost any power division.

The experimental results show that, centered at 55.6 kmc, the frequency range is larger than 20 per cent. Adjusted for 3 db division with the transducers from $TE_{01}^{\circledcirc}$ to $TE_{10}^{\square}$ included, the balance is better than 0.5 db and the unwanted reflections in the driven and balanced (isolation) arms are at least 23 db below the input signal.

No efforts have been made to improve either the hybrid or the transducers. The possible changes for the hybrid are of an experimental nature and consist in varying the diameter of the gap region and including circular symmetric lumped discontinuities to improve the balance and decrease the unwanted reflections. The possible improvement of the transducer consists in passing from the relatively simple-to-build linear

taper used in these experiments to more sophisticated designs[10] that reduce mode conversion.

Using the hybrid together with band-reflection, band-transmission or cutoff waveguides, it is possible to build low-loss constant-resistance channel-dropping filters. In particular, the use of cutoff waveguides permits us to design filters with almost rectangular transfer characteristics.

Hybrids and filters described in this paper operate with circular-electric modes, but their equivalents operating with TE modes in rectangular waveguides can be easily derived by the reader. The design of $TE_{10}^{\square}$ mode conversion filters is given in an accompanying paper.[4]

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